Homework #6

Chapter 20

The Nucleus: A Chemist View

1. a) Thermodynamic Stability: The potential energy of a particular nucleus compared to the sum of the potential energies of its component protons and neutrons.
   b) Kinetic Stability: The probability that a nucleus will undergo decomposition to form a different nucleus.
   c) Radio Active Decay: A spontaneous decomposition of a nucleus to form a different nucleus.
   d) β-particle Production: A decay process for radioactive nuclides where an electron is produced; the mass number remains constant and the atomic number changes. (β-particle= ${}_{-1}^{0}e$)
   e) α-particle Production: A common mode of decay for heavy radioactive nuclides where a helium nucleus is produced, causing the atomic number and the mass number to change. (α-particle=${}_{2}^{4}He$)
   f) Positron Production: A mode of nuclear decay in which a particle is formed having the same mass as an electron but opposite in charge. (positron= ${}_{1}^{0}e$)
   g) Electron Capture: A process in which one of the inner-orbital electrons in an atom is captured by the nucleus.
   h) γ-ray Emission: The production of high-energy photons (gamma rays) that frequently accompany nuclear decays and particle reactions.(γ-ray= γ)

6. a) ${}_{31}^{73}Ga \rightarrow {}_{32}^{73}Ge + {}_{-1}^{0}e$
   b) ${}_{78}^{192}Pt \rightarrow {}_{76}^{188}Os + {}_{2}^{4}He$
   c) ${}_{83}^{205}Bi \rightarrow {}_{82}^{205}Pb + {}_{1}^{0}e$
   d) ${}_{96}^{241}Cm + {}_{-1}^{0}e \rightarrow {}_{95}^{241}Am$
   e) ${}_{27}^{60}Co \rightarrow {}_{28}^{60}Ni + {}_{-1}^{0}e$
   f) ${}_{43}^{97}Tc + {}_{-1}^{0}e \rightarrow {}_{42}^{97}Mo$
   g) ${}_{43}^{99}Tc \rightarrow {}_{44}^{99}Ru + {}_{-1}^{0}e$
   h) ${}_{94}^{239}Pu \rightarrow {}_{92}^{235}U + {}_{2}^{4}He$

7. a) ${}_{31}^{68}Ga + {}_{-1}^{0}e \rightarrow {}_{30}^{68}Zn$
   b) ${}_{29}^{62}Cu \rightarrow {}_{28}^{62}Ni + {}_{1}^{0}e$
   c) ${}_{87}^{212}Fr \rightarrow {}_{85}^{208}At + {}_{2}^{4}He$
   d) ${}_{51}^{129}Sb \rightarrow {}_{52}^{129}Te + {}_{-1}^{0}e$

9. ${}_{97}^{247}Bk \rightarrow {}_{82}^{207}Pb + x\,{}_{2}^{4}He + y\,{}_{-1}^{0}e$

   The mass number changes by 40 (247-207=40) only the α–particle can change the mass number. Therefore, y=10, in order to get mass number to change by 40. When 10 α–particles are added the total, protons on the product side (from the α-particles and the lead) are (82+20=102) and to equal the reactants they must be 97. Therefore, (102-97=5) y=5 and 5 β-particles must be emitted.

13. If flourine-19 is the only stable nuclei, compare the other nuclei to fluorine-19 to determine if there are too many neutrons or protons.
Fluorine-21, too many neutrons, therefore decays by β–particle production.
Fluorine -18, too many protons, therefore, decays by either positron production or electron capture. (α-particle production only happens for large nuclei).
Fluorine-17, too many protons, therefore, decays by either positron production or electron capture. (α-particle production only happens for large nuclei).

17. The question is asking for you to find the rate (particles per second).

$$Rate = kN$$

What we know

$$t_{1/2} = \frac{ln(2)}{k}$$

$$Rate = kN$$

$$t_{1/2} = 433\ y \left(\frac{365\ d}{1\ y}\right)\left(\frac{24\ h}{1\ d}\right)\left(\frac{60\ m}{1\ h}\right)\left(\frac{60\ s}{1\ m}\right) = 1.366 \times 10^{10}\ s$$

$$N = 5.00\ g \left(\frac{1\ mol\ Am}{243\ g\ Am}\right)\left(\frac{6.022\times10^{23}\ particles}{1\ mol}\right) = 1.24 \times 10^{22}\ particles$$

Determine k

$$k = \frac{ln(2)}{t_{1/2}} = \frac{ln(2)}{1.363 \times 10^{10}\ s} = 5.085 \times 10^{-11}\ \frac{1}{s}$$

Determine rate

$$Rate = kN = \left(5.085 \times 10^{-11}\ \tfrac{1}{s}\right)(1.24 \times 10^{22}\ particles)$$

$$Rate = 6.34 \times 10^{11}\ \frac{particles}{s}$$

6.34×10^{11} decays will happen per second.

18. The longer the half-life the more stable the isotope, therefore, Kr-81 is the most stable. The “hotter” (least stable) the isotope the shorter the half-life, therefore, Kr-73 is the hottest isotope.
What we know

$$t_{1/2} = \frac{ln(2)}{k}$$

$$ln(N) = -kt + ln(N_\circ)$$

$$ln\left(\frac{N}{N_\circ}\right) = -kt$$

For all cases they are interested in the time it takes for 87.5% to decay.
Therefore, if $N_\circ$=1 than N=1-0.875=0.125

$$ln\left(\frac{0.125}{1}\right) = -kt$$

$$-2.08 = -kt$$

Kr-73

Find k

$$k = \frac{ln(2)}{t_{1/2}} = \frac{ln(2)}{27\ s} = 0.026\ \frac{1}{s}$$

Find t

$$t = \frac{2.08}{k} = \frac{2.08}{0.026\,\frac{1}{s}} = 80.\,s$$

Kr-74

Find k

$$k = \frac{ln(2)}{t_{1/2}} = \frac{ln(2)}{11.5\,m} = 0.0603\,\frac{1}{m}$$

Find t

$$t = \frac{2.08}{k} = \frac{2.08}{0.0603\,\frac{1}{m}} = 34.5\,m$$

Kr-76

Find k

$$k = \frac{ln(2)}{t_{1/2}} = \frac{ln(2)}{14.8\,h} = 0.0468\,\frac{1}{h}$$

Find t

$$t = \frac{2.08}{k} = \frac{2.08}{0.0468\,\frac{1}{h}} = 44.4\,h$$

Kr-81

Find k

$$k = \frac{ln(2)}{t_{1/2}} = \frac{ln(2)}{2.1 \times 10^{5}\,y} = 3.3 \times 10^{-6}\,\frac{1}{y}$$

Find t

$$t = \frac{2.08}{k} = \frac{2.08}{3.3 \times 10^{-9}\,\frac{1}{y}} = 6.3 \times 10^{5}\,y$$

21. What we know

$$t_{1/2} = \frac{ln(2)}{k}$$

$$ln(N) = -kt + ln(N_{\circ})$$

$$ln\left(\frac{N}{N_{\circ}}\right) = -kt$$

We need to calculate $\left(\frac{N}{N_{\circ}}\right)$

Determine K

$$k = \frac{ln(2)}{t_{1/2}} = \frac{ln(2)}{28.8\,y} = 0.0241\,\frac{1}{y}$$

Determine $\left(\frac{N}{N_{\circ}}\right)$

From July 16, 1945 to July 16, 2016 is 71 years

$$\left(\frac{N}{N_{\circ}}\right) = e^{-kt} = e^{-\left(0.0241\,\frac{1}{y}\right)(71\,y)} = 0.181$$

18.1% of the original strontium-90 remains as of July 16, 20016

29. What we know

$$t_{1/2} = \frac{ln(2)}{k}$$
$$ln(N) = -kt + ln(N_\circ)$$
$$ln\left(\frac{N}{N_\circ}\right) = -kt$$

If 17% of ^{3}H is needed to read the watch dial we are interested in the time until

$$\left(\frac{N}{N_\circ}\right) = 0.17$$

Calculate k

$$k = \frac{ln(2)}{t_{1/2}} = \frac{ln(2)}{12.3\ y} = 0.0564\,\frac{1}{y}$$

Calculate t

$$t = \frac{ln\left(\frac{N}{N_\circ}\right)}{k} = \frac{ln(0.17)}{0.0564\,\frac{1}{y}} = 31\ y$$

Therefore, the watch could be read in the dark until 1975 (1944+31)

34. a) In order to use a substance for dating purposes the substance can only come from the radioactive decay and not from other sources. ^{40}Ca comes from other sources in addition to the decay, therefore, it cannot be used for dating purposes.

b) The assumption has to be made that all of the ^{40}Ar is coming from the decay of ^{40}K and none of the ^{40}Ar is lost.

c) $\frac{^{40}Ar}{^{40}K} = 0.95$

If mass of ^{40}Ar was 0.95 g then the mass of ^{40}K would have to be 1.00 g.
Based on these numbers, determine the original mass of ^{40}K.
Only 10.7% of the ^{40}K will decay to ^{40}Ar. The mass of 1 atom of ^{40}Ar is approximately equal to the mass of 1 atom of ^{40}K.
Therefore,

$$m_{original(^{40}K)}(0.107) = m_{final(^{40}\,Ar)}$$
$$m_{original(^{40}K)}(0.107) = 0.95\ g$$
$$m_{original(^{40}K)} = 8.9\ g$$

Calculate k

$$k = \frac{ln(2)}{t_{1/2}} = \frac{ln(2)}{1.17 \times 10^{-10}\ y} = 5.46 \times 10^{-10}\,\frac{1}{y}$$

Calculate the age of the rock t

$$t = \frac{ln\left(\frac{N}{N_\circ}\right)}{k} = \frac{ln\left(\frac{1.00\ g}{8.9\ g}\right)}{5.46 \times 10^{-10}\,\frac{1}{y}} = 4.00 \times 10^{9}\ y$$

d) If some of the ^{40}Ar has escaped, the rock will look younger than it actually is.

35. $\Delta E = \Delta mc^2$

$$\Delta m = \frac{\Delta E}{c^2} = \frac{-3.9 \times 10^{23}\,\frac{J}{s}}{\left(3.00 \times 10^8\,\frac{m}{s}\right)^2} = -4.3 \times 10^6\,\frac{kg}{s}$$

Since the sun is giving off energy the ΔE is negative. Δm is negative because mass is being lost at a rate of 4.3×10^6 kg per s.

37. The binding energy results from the difference in mass between the mass of the protons neutrons and electrons separately and the mass of the overall atom

$$\Delta m = m_{^{56}Fe} - 26m_{^1H} - 30m_n$$

$$\Delta m = 55.9349\,u - 26(1.0078\,u) - 30(1.0087\,u) = -0.5289\,u$$

The negative sign represents that mass is being lost which will result in energy being released.

Change the weight to kg

$$-0.5289\,u\left(\frac{1.6605\times10^{-27}\,kg}{1\,u}\right) = -8.78 \times 10^{-28}\,kg$$

Calculate the nuclear binding energy (ΔE)

$$\Delta E = \Delta mc^2 = (8.78 \times 10^{-28}\,kg)\left(2.9979 \times 10^8\,\frac{m}{s}\right)^2 = 7.89 \times 10^{-11}\,J$$

Calculate the binding energy per nucleon

$$\frac{7.89 \times 10^{-11}\,J}{56} = 1.41 \times 10^{-12}\,\frac{J}{nucleon}$$

41. Calculate the mass of protons, neutrons, and electrons in ^{27}Mg if they were all separate in kg.

$$m = 12m_{^1H} + 15m_n = 12(1.0078\,u) - 15(1.0087\,u) = 27.2241\,u$$

Calculate the nuclear binding energy per atom of ^{27}Mg

$$-1.326 \times 10^{-12}\,\frac{J}{nucleon}(27\,nucleon) = -3.580 \times 10^{-11}\,J$$

Calculate the change in mass, in u, that is needed to produce 3.580×10^{-11} J

$$\Delta E = \Delta mc^2$$

$$\Delta m = \frac{\Delta E}{c^2} = \frac{-3.580 \times 10^{-11}\;J}{\left(2.99792 \times 10^8\,\frac{m}{s}\right)^2} = -3.983 \times 10^{-28}\,kg$$

$$-3.983 \times 10^{-28}\,kg\left(\frac{1\,u}{1.6605\times10^{-27}\,kg}\right) = -0.23\,amu$$

Calculate the molar mass of the atom

$$\Delta m = m_{atom} - m_{atom(particles\ seperated)}$$

$$m_{atom} = \Delta m + m_{atom(particles\ seperated)}$$

$$m_{atom} = -0.23\,u + 27.22441\,u = 26.99\,u$$

47. Fission: Splitting of a heavy nucleus into two (or more) lighter nuclei.

Fusion: Combining two light nuclei to form a heavier nucleus.

The nuclear binding energy increases as the mass number increases until you get to ^{56}Fe. This results in fusion processes being energetically favorable for atoms that have an atomic number of less than 26. After you get to ^{56}Fe the nuclear binding energy decreases as you increase mass number, making fission energetically more favorable for atoms with an atomic number greater than 26. See figure 20.10 in text.

53. Radioactive nuclides can be used to see if equilibrium is a dynamic process by setting up an equilibrium in which either the products or reactants contain a radioactive nuclide. The system can then be monitored to determine if the radioactive nuclides stay bound to the same atoms they were bound with originally or switch the atoms that they are bound to. If they switch atoms that they are bound to then equilibrium is a dynamic process. If they stay bound to the same atoms then equilibrium is a steady state.

56. Radiotracer: A radioactive nuclide introduced into an organism for diagnostic purposes whose pathway can be traced by monitoring its radioactivity. ^{14}C and ^{32}P work well as radiotracers because the molecules in the body contain carbon and/or phosphorus; they will be incorporated into the worker molecules of the body easily, which allows monitoring of the pathways of these worker molecules.

61. Calculate the Δm (electron and positron weights are the same)
$\Delta m = 0 - 2(9.10939 \times 10^{-31}\ kg) = -1.82188 \times 10^{-30}\ kg$
Calculate the energy

$$\Delta E = \Delta mc^2 = (-1.82188 \times 10^{-30}\ kg)\left(2.99792 \times 10^8\ \frac{m}{s}\right)^2$$
$$= -1.63742 \times 10^{-13}\ J$$

Therefore the energy released per photon is $\frac{1.63742 \times 10^{-13}\ J}{2} = -8.18710 \times 10^{-14}\ J$
Calculate the wavelength of the photons

$$E = \frac{hc}{\lambda}$$
$$\lambda = \frac{hc}{E} = \frac{(6.62608 \times 10^{-34}\ J)\left(2.99792 \times 10^8\ \frac{m}{s}\right)}{8.18710 \times 10^{-14}\ J} = 2.42631 \times 10^{-12}\ m$$
$$\lambda = 2.42631 \times 10^{-3}\ nm$$

77. They want you to calculate the percentages of ^{238}U and ^{235}U when the earth was formed.
Assume that currently you have a sample with 10,000 U atoms. Therefore, currently (in this problem the final number) you would have 9928 atoms of ^{238}U and 72 atoms of ^{235}U is 72.
Calculate the initial amount of each species using:

$$ln(N) = -kt + ln(N_\circ)$$

Find K for ^{238}U

$$k = \frac{ln(2)}{t_{1/2}} = \frac{ln(2)}{4.5 \times 10^9\ y} = 1.5 \times 10^{-10}\ \frac{1}{y}$$

Find initial amount of ^{238}U

$$ln(9928) = -\left(1.5 \times 10^{-10}\ \frac{1}{y}\right)(4.5 \times 10^9\ y) + ln(N_\circ)$$
$$N_\circ = 19{,}000$$

Find K for ^{235}U

$$k = \frac{ln(2)}{t_{1/2}} = \frac{ln(2)}{7.1 \times 10^8\ y} = 9.8 \times 10^{-10}\ \frac{1}{y}$$

Find initial amount of ^{235}U

$$ln(72) = -\left(9.8 \times 10^{-10}\,\frac{1}{y}\right)(4.5 \times 10^{9}\ y) + ln(N_\circ)$$

$$N_\circ = 5{,}900$$

Determine percentage of ^{238}U

$$U_{tot} = 5{,}900 + 19{,}000 = 25{,}000$$

$$\%\ 238_U = \frac{19{,}000}{25{,}000} = 76\%$$

Determine percentage of ^{235}U

$$\%\ 235_U = \frac{5{,}900}{25{,}000} = 24\%$$

80. Determine the missing particle

$$^{58}_{26}Fe + 2\,^{1}_{0}n \rightarrow {}^{60}_{27}Co + ?$$

The missing particle must be $_{-1}^{\ 0}e$ a beta particle/electron

Determine the binding energy of ^{60}Co

$$\Delta m = m_{60_{Co}} - 27m_{1_H} - 33m_n$$

$$\Delta m = 59.9338\ amu - 27(1.0078\ u) - 33(1.0087\ u) = -0.5639\ u$$

The negative sign represents that mass is being lost which will result in energy being released.

Change the weight to kg

$$-0.5639\ u\left(\frac{1.6605\times10^{-27}\ kg}{1\ u}\right) = -9.364 \times 10^{-28}\ kg$$

Calculate the nuclear binding energy (ΔE)

$$\Delta E = \Delta mc^2 = (9.364 \times 10^{-28}\ kg)\left(2.99792 \times 10^{8}\ \frac{m}{s}\right)^2 = 8.416 \times 10^{-11}\ J$$

Calculate the binding energy per nucleon

$$\frac{8.416 \times 10^{-11}\ J}{60} = 1.403 \times 10^{-12}\ \frac{J}{nucleon}$$

Determine the de Broglie wavelength of the ejected e^-

$$\lambda = \frac{h}{mv} = \frac{6.62608 \times 10^{-34}\ J \cdot s}{(9.10939 \times 10^{-31}\ kg)\left(0.90\left(2.9979246 \times 10^{8}\ \frac{m}{s}\right)\right)}$$

$$= 2.696 \times 10^{-12}\ m$$